

3-2

Solving Systems of Equations Algebraically

Main Ideas

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

New Vocabulary

substitution method
elimination method

GET READY for the Lesson

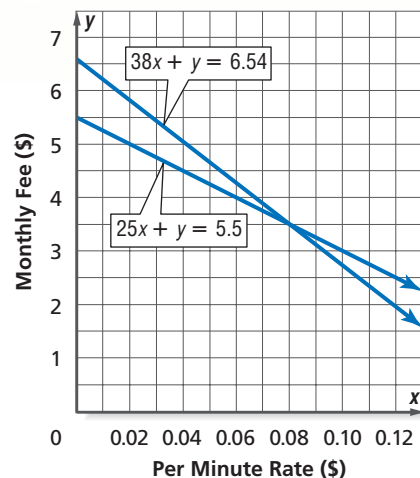
In January, Yolanda's long-distance bill was \$5.50 for 25 minutes of calls. The bill was \$6.54 in February, when Yolanda made 38 minutes of calls. What are the rate per minute and flat fee the company charges?

Let x equal the rate per minute, and let y equal the monthly fee.

January bill: $25x + y = 5.5$

February bill: $38x + y = 6.54$

Sometimes it is difficult to determine the exact coordinates of the point where the lines intersect from the graph. For systems of equations like this one, it may be easier to solve the system by using algebraic methods.



Substitution One algebraic method is the **substitution method**. Using this method, one equation is solved for one variable in terms of the other. Then, this expression is substituted for the variable in the other equation.

EXAMPLE Solve by Using Substitution

1 Use substitution to solve the system of equations.

$$x + 2y = 8$$

$$\frac{1}{2}x - y = 18$$

Solve the first equation for x in terms of y .

$$x + 2y = 8$$

First equation

$$x = 8 - 2y$$

Subtract $2y$ from each side.

Substitute $8 - 2y$ for x in the second equation and solve for y .

$$\frac{1}{2}x - y = 18$$

Second equation

$$\frac{1}{2}(8 - 2y) - y = 18$$

Substitute $8 - 2y$ for x .

$$4 - y - y = 18$$

Distributive Property

$$-2y = 14$$

Subtract 4 from each side.

$$y = -7$$

Divide each side by -2 .

Study Tip

Coefficient of 1

It is easier to solve for the variable that has a coefficient of 1.

(continued on the next page)

Now, substitute the value for y in either original equation and solve for x .

$$x + 2y = 8 \quad \text{First equation}$$

$$x + 2(-7) = 8 \quad \text{Replace } y \text{ with } -7.$$

$$x - 14 = 8 \quad \text{Simplify.}$$

$$x = 22$$

The solution of the system is $(22, -7)$.

CHECK Your Progress

1A. $2x - 3y = 2$

$$x + 2y = 15$$

1B. $7y = 26 + 11x$

$$x - 3y = 0$$

STANDARDIZED TEST EXAMPLE

Solve by Substitution

- 2** Matthew stopped for gasoline twice on a long car trip. The price of gasoline at the first station where he stopped was \$2.56 per gallon. At the second station, the price was \$2.65 per gallon. Matthew bought a total of 36.1 gallons of gasoline and spent \$94.00. How many gallons of gasoline did Matthew buy at the first gas station?

A 17.6

B 18.5

C 19.2

D 20.1

Read the Item

You are asked to find the number of gallons of gasoline that Matthew bought at the first gas station.

Solve the Item

- Step 1** Define variables and write the system of equations. Let x represent the number of gallons bought at the first station and y represent the number of gallons bought at the second station.

$$x + y = 36.1 \quad \text{The total number of gallons was 36.1.}$$

$$2.56x + 2.65y = 94 \quad \text{The total price was \$94.}$$

- Step 2** Solve one of the equations for one of the variables in terms of the other. Since the coefficient of y is 1 and you are asked to find the value of x , it makes sense to solve the first equation for y in terms of x .

$$x + y = 36.1 \quad \text{First equation}$$

$$y = 36.1 - x \quad \text{Subtract } x \text{ from each side.}$$

- Step 3** Substitute $36.1 - x$ for y in the second equation.

$$2.56x + 2.65y = 94 \quad \text{Second equation}$$

$$2.56x + 2.65(36.1 - x) = 94 \quad \text{Substitute } 36.1 - x \text{ for } y.$$

$$2.56x + 95.665 - 2.65x = 94 \quad \text{Distributive Property}$$

$$-0.09x = -1.665 \quad \text{Simplify.}$$

$$x = 18.5 \quad \text{Divide each side by } -0.09.$$

Test-Taking Tip

Even if the question does not ask you for both variables, it is still a good idea to find both so that you can check your answer.

Step 4 Matthew bought 18.5 gallons of gasoline at the first gas station. The answer is B.

CHECK Your Progress

2. **COMIC BOOKS** Dante spent \$11.25 on 3 new and 4 old comic books, and Samantha spent \$15.75 on 10 old and 3 new ones. If comics of one type are sold at the same price, what is the price in dollars of a new comic book?

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Elimination Another algebraic method is the **elimination method**. Using this method, you eliminate one of the variables by adding or subtracting the equations. When you add two true equations, the result is a new equation that is also true.

EXAMPLE Solve by Using Elimination

 Use the elimination method to solve the system of equations.

$$4a + 2b = 15$$

$$2a + 2b = 7$$

In each equation, the coefficient of b is 2. If one equation is subtracted from the other, the variable b will be eliminated.

$$4a + 2b = 15$$

$$(-) 2a + 2b = 7$$

$$2a = 8 \quad \text{Subtract the equations.}$$

$$a = 4 \quad \text{Divide each side by 2.}$$

Now find b by substituting 4 for a in either original equation.

$$2a + 2b = 7 \quad \text{Second equation}$$

$$2(4) + 2b = 7 \quad \text{Replace } a \text{ with 4.}$$

$$8 + 2b = 7 \quad \text{Multiply.}$$

$$2b = -1 \quad \text{Subtract 8 from each side.}$$

$$b = -\frac{1}{2} \quad \text{Divide each side by 2.}$$

The solution is $\left(4, -\frac{1}{2}\right)$.

CHECK Your Progress

3A. $2x + y = 4$

$$3x + y = 8$$

3B. $5b = 20 + 2a$

$$2a + 4b = 7$$

Study Tip

Alternative Method

You may find it confusing to subtract equations. It may be helpful to multiply the second equation by -1 and then add the equations.

Sometimes, adding or subtracting the two equations will not eliminate either variable. You may use multiplication to write an equivalent equation so that one of the variables has the same or opposite coefficient in both equations. When you multiply an equation by a nonzero number, the new equation is equivalent to the original equation.

EXAMPLE Multiply, Then Use Elimination

- 4 Use the elimination method to solve the system of equations.

$$3x - 7y = -14$$

$$5x + 2y = 45$$

Multiply the first equation by 2 and the second equation by 7. Then add the equations to eliminate the y variable.

$$3x - 7y = -14$$

Multiply by 2.

$$6x - 14y = -28$$

$$5x + 2y = 45$$

Multiply by 7.

$$(+)\ 35x + 14y = 315$$

$$\begin{array}{r} 41x \\ = 287 \end{array}$$

Add the equations.

$$x = 7$$

Divide each side by 41.

Replace x with 7 and solve for y .

$$3x - 7y = -14 \quad \text{First equation}$$

$$3(7) - 7y = -14 \quad \text{Replace } x \text{ with 7.}$$

$$21 - 7y = -14 \quad \text{Multiply.}$$

$$-7y = -35 \quad \text{Subtract 21 from each side.}$$

$$y = 5 \quad \text{Divide each side by } -7.$$

The solution is $(7, 5)$.

CHECK Your Progress

4A. $3x + 4y = 14$

$$4x + 5y = 17$$

4B. $2x - 4y = 28$

$$4x = 17 - 5y$$

If you add or subtract two equations in a system and the result is an equation that is never true, then the system is inconsistent. If the result when you add or subtract two equations in a system is an equation that is always true, then the system is dependent.

EXAMPLE Inconsistent System

- 5 Use the elimination method to solve the system of equations.

$$8x + 2y = 17$$

$$-4x - y = 9$$

Use multiplication to eliminate x .

$$8x + 2y = 17$$

$$8x + 2y = 17$$

$$-4x - y = 9$$

Multiply by 2.

$$-8x - 2y = 18$$

$$0 = 35 \quad \text{Add the equations.}$$

Since there are no values of x and y that will make the equation $0 = 35$ true, there are no solutions for this system of equations.

CHECK Your Progress

5A. $8y = 2x + 48$

$$y - \frac{1}{4}x = 6$$

5B. $x - 0.5y = -3$

$$2x - y = 6$$

Study Tip**Alternative Method**

You could also multiply the first equation by 5 and the second equation by 3. Then subtract to eliminate the x variable.

Example 1
(pp. 123–124)

Solve each system of equations by using substitution.

1. $y = 3x - 4$

$y = 4 + x$

3. $a - b = 2$

$-2a + 3b = 3$

2. $4c + 2d = 10$

$c + 3d = 10$

4. $3g - 2h = -1$

$4g + h = 17$

Example 2
(pp. 124–125)

5. **STANDARDIZED TEST PRACTICE** Campus Rentals rents 2- and 3-bedroom apartments for \$700 and \$900 per month, respectively. Last month they had six vacant apartments and reported \$4600 in lost rent. How many 2-bedroom apartments were vacant?

A 2

B 3

C 4

D 5

Examples 3–5
(pp. 125–126)

Solve each system of equations by using elimination.

6. $2r - 3s = 11$

$2r + 2s = 6$

9. $\frac{1}{4}x + y = \frac{11}{4}$
 $x - \frac{1}{2}y = 2$

7. $5m + n = 10$

$4m + n = 4$

10. $\frac{1}{6}y - 2 = \frac{1}{9}$
 $12 = 18y$

8. $2p + 4q = 18$

$3p - 6q = 3$

11. $1.25x - y = -7$
 $4y = 5x + 28$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–17	1, 2
18–21	3, 4
22, 23	5

Solve each system of equations by using substitution.

12. $2j - 3k = 3$

$j + k = 14$

15. $-w - z = -2$

$4w + 5z = 16$

13. $2r + s = 11$

$6r - 2s = -2$

16. $3s + 2t = -3$

$s + \frac{1}{3}t = -4$

14. $5a - b = 17$

$3a + 2b = 5$

17. $2x + 4y = 6$

$7x = 4 + 3y$

Solve each system of equations by using elimination.

18. $u + v = 7$

$2u + v = 11$

21. $4x - 5y = 17$

$3x + 4y = 5$

19. $m - n = 9$

$7m + n = 7$

22. $2c + 6d = 14$

$-\frac{7}{3} + \frac{1}{3}c = -d$

20. $r + 4s = -8$

$3r + 2s = 6$

23. $6d + 3f = 12$

$2d = 8 - f$

SKIING For Exercises 24 and 25, use the following information.

All 28 members in Crestview High School's Ski Club went on a one-day ski trip. Members can rent skis for \$16 per day or snowboards for \$19 per day. The club paid a total of \$478 for rental equipment.

24. Write a system of equations that represents the number of members who rented the two types of equipment.

25. How many members rented skis and how many rented snowboards?

INVENTORY For Exercises 26 and 27, use the following information.

Beatriz is checking a shipment of technology equipment that contains laser printers that cost \$700 each and color monitors that cost \$200 each. She counts 30 boxes on the loading dock. The invoice states that the order totals \$15,000.

26. Write a system of two equations that represents the number of each item.

27. How many laser printers and how many color monitors were delivered?

Cross-Curricular Project



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EXTRA PRACTICE
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H.O.T. Problems

Solve each system of equations by using either substitution or elimination.

28. $3p - 6q = 6$

$2p - 4q = 4$

31. $6g - 8h = 50$

$6h = 22 - 4g$

34. $3u + 5v = 6$

$2u - 4v = -7$

37. $8 = 0.4m + 1.8n$

$1.2m + 3.4n = 16$

29. $10m - 9n = 15$

$5m - 4n = 10$

32. $2p = 7 + q$

$6p - 3q = 24$

35. $3a = -3 + 2b$

$3a + b = 3$

38. $s + 3t = 27$

$2t = 19 - \frac{1}{2}s$

30. $3c - 7d = -3$

$2c + 6d = -34$

33. $3x = -31 + 2y$

$5x + 6y = 23$

36. $0.25x + 1.75y = 1.25$

$-0.5x + 2 = 2.5y$

39. $2f + 2g = 18$

$\frac{1}{6}f + \frac{1}{3}g = 1$

TEACHING For Exercises 40–42, use the following information.

Mr. Talbot is writing a science test. It will have true/false questions worth 2 points each and multiple-choice questions worth 4 points each for a total of 100 points. He wants to have twice as many multiple-choice questions as true/false.

40. Write a system of equations that represents the number of each type of question.

41. How many of each type of question will be on the test?

42. If most of his students can answer true/false questions within 1 minute and multiple-choice questions within $1\frac{1}{2}$ minutes, will they have enough time to finish the test in 45 minutes?

EXERCISE For Exercises 43 and 44, use the following information.

Megan exercises every morning for 40 minutes. She does a combination of step aerobics, which burns about 11 Calories per minute, and stretching, which burns about 4 Calories per minute. Her goal is to burn 335 Calories during her routine.

43. Write a system of equations that represents Megan's morning workout.

44. How long should she do each activity in order to burn 335 Calories?

45. **OPEN ENDED** Give a system of equations that is more easily solved by substitution and a system of equations that is more easily solved by elimination.

46. **REASONING** Make a conjecture about the solution of a system of equations if the result of subtracting one equation from the other is $0 = 0$.

47. **FIND THE ERROR** Juanita and Jamal are solving the system $2x - y = 6$ and $2x + y = 10$. Who is correct? Explain your reasoning.

Juanita

$$\begin{array}{r} 2x - y = 6 \\ (-)2x + y = 10 \\ \hline 0 = -4 \end{array}$$

The statement $0 = -4$ is never true, so there is no solution.

Jamal

$$\begin{array}{rcl} 2x - y = 6 & 2x - y = 6 \\ (+)2x + y = 10 & 2(4) - y = 6 \\ \hline 4x = 16 & 8 - y = 6 \\ x = 4 & y = 2 \end{array}$$

The solution is $(4, 2)$.

48. **CHALLENGE** Solve the system of equations.

$\frac{1}{x} + \frac{3}{y} = \frac{3}{4}$ (Hint: Let $m = \frac{1}{x}$ and $n = \frac{1}{y}$.)

$\frac{3}{x} - \frac{2}{y} = \frac{5}{12}$

49. **Writing in Math** Use the information on page 123 to explain how a system of equations can be used to find a flat fee and a per-unit rate. Include a solution of the system of equations in the application at the beginning of the lesson.

STANDARDIZED TEST PRACTICE

50. **ACT/SAT** In order to practice at home, Tadeo purchased a basketball and a volleyball that cost a total of \$67, not including tax. If the price of the basketball b is \$4 more than twice the cost of the volleyball v which system of linear equations could be used to determine the price of each ball?
- A $b + v = 67$
 $b = 2v - 4$
- B $b + v = 67$
 $b = 2v + 4$
- C $b + v = 4$
 $b = 2v - 67$
- D $b + v = 4$
 $b = 2v + 67$
51. **REVIEW** The caterer at a brunch bought several pounds of chicken salad and several pounds of tuna salad. The chicken salad cost \$9 per pound, and the tuna salad cost \$6 per pound. He bought a total of 14 pounds of salad and paid a total of \$111. How much chicken salad did he buy?
- F 6 pounds
- G 7 pounds
- H 8 pounds
- J 9 pounds

Spiral Review

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*. (Lesson 3-1)

52. $y = x + 2$
 $y = x - 1$
53. $4y - 2x = 4$
 $y - \frac{1}{2}x = 1$
54. $3x + y = 1$
 $y = 2x - 4$

Graph each inequality. (Lesson 2-7)

55. $x + y \leq 3$
56. $5y - 4x < -20$
57. $3x + 9y \geq -15$

Write each equation in standard form. Identify A , B , and C . (Lesson 2-2)

58. $y = 7x + 4$
59. $x = y$
60. $3x = 2 - 5y$
61. $6x = 3y - 9$
62. $y = \frac{1}{2}x - 3$
63. $\frac{2}{3}y - 6 = 1 - x$

64. **ELECTRICITY** Find the amount of current I (in amperes) produced if the electromotive force E is 1.5 volts, the circuit resistance R is 2.35 ohms, and the resistance r within a battery is 0.15 ohms, using the formula

$$I = \frac{E}{R + r} \quad (\text{Lesson 1-1})$$

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether the given point satisfies each inequality. (Lesson 2-7)

65. $3x + 2y \leq 10$; $(2, -1)$
66. $4x - 2y > 6$; $(3, 3)$
67. $7x + 4y \geq -15$; $(-4, 2)$